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A model for string-breaking in QCD

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Abstract: We present a model for string breaking based on the existence of chromoelectric flux tubes. We predict the form of the long-range potential and obtain an estimate of the string breaking length. A prediction is also obtained for the behaviour with temperature of the string breaking length near the deconfinement phase transition. We plan to use this model as a guide for a program of study of string breaking on the lattice.

Keywords: Nonperturbative Effects, Confinement, Lattice Gauge Field Theories, Phenomenological Models.

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Contents

1. Introduction

In pure gauge theories, the potential energy of a static $\bar{q}q$ pair grows with the quark-antiquark distance r as

$$V(r) = \sigma r \tag{1.1}$$

where σ is known as string tension. From Regge phenomenology, $\sigma \simeq (440\,\mathrm{MeV})^2$. Equation (1.1) is confirmed by numerical simulations on the lattice, see e.g. [1], or [2] and references therein for recent calculations.

In full QCD, due to the presence of dynamical quarks, string breaking is expected to occur. Equation (1.1) yields an accurate description of the potential up to some distance, above which the system prefers to convert the energy into $\bar{q}q$ pairs, or mesons. String breaking has been observed on the lattice, see e.g. [3]–[6], and its behaviour at finite temperature around the deconfining phase transition has been studied numerically [7].

In this paper, we construct a dynamical model of string breaking. Lattice simulations have shown that the chromoelectric field in the presence of a static quark-antiquark pair looks like an Abrikosov flux tube [8] of transverse size $r_\perp$, with $r_\perp \simeq 0.2 \div 0.3\,\mathrm{fm}$ [9]–[14], as conjectured since a long time [15]. This picture is consistent with a dual superconductor mechanism of confinement [16, 17], the superconductor being at the border between type-I and type-II. We shall interpret string breaking as due to the production of light quark-antiquark pairs by the chromoelectric field of the dual Abrikosov-Nielsen-Olesen flux tube. We plan to verify this idea by an extended program of lattice simulations. In this paper, we discuss the general features of the model and present some analytic calculations in limiting cases.

A similar phenomenon occurs in electrodynamics when one considers the potential inside a condenser. When the distance r between the plates is increased, at constant electric field, the energy increases linearly with r, up to some distance where the system becomes unstable because of e^+e^- pair production.

For a constant static electric field the rate of production of spin-1/2 particle pairs per unit volume and unit time is given by [18]:

$$w_{\rm QED} = 2\frac{(eE)^2}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{e^{-\pi n m^2/eE}}{n^2}\,. \qquad (1.2)$$

In QCD the analog of the condenser is the flux tube, electron-positron pairs are replaced by quark-antiquark pairs, the coupling constant e by the strong coupling constant g, and a factor $N_c N_f$ must be added to properly take into account the number of degrees of freedom. Hence,

$$w_{\rm QCD} \equiv w = 2N_c N_f\frac{(g\mathcal{E})^2}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{e^{-\pi n m^2/g\mathcal{E}}}{n^2}\,, \qquad (1.3)$$

where $\mathcal{E}$ is a constant static chromoelectric field. However, free quarks do not exist in QCD, and therefore m in this formula is some mass parameter to be determined, which can be either the constituent quark mass, or a typical hadron mass (e.g., m_π). On the lattice, quark masses can be changed at will, and m can be understood as a function of them.

In this work, in order to understand the relevant physics, we shall study the simplified case, in which the chromoelectric field is averaged over the transverse direction and then considered as uniform in space. A partial physical justification is that, due to the higher modes of the string, the tube configuration will fluctuate with a frequency that is much higher than the rate of pair production. More precise calculations taking into account the transverse size of the flux tube and the space dependence of the electric field can be performed and will be presented elsewhere. The use of eq. (1.3), which relies on the assumption that the field is constant in space, will in any case be a good approximation if the transverse size of the tube is larger than $1/m$. The volume Ω that we consider is the physical three-dimensional volume of the flux tube, of cross section S, multiplied by a time equal to the length of the tube r in natural units. This is a natural choice. Indeed, in order to measure the force between the heavy $\bar{q}q$ pair, we need at least the time to propagate the interaction between the two particles. Also this choice can be tested on the lattice. The interaction can be described by an elastic potential if no pair has been produced in the meantime. Hence, the long-range part of the potential is given by:

$$V(r) = \sigma r\, e^{-r^2 S w} \qquad (1.4)$$

i.e. the linear potential σr multiplied by the probability that no pairs are created in the volume Ω. Clearly, $V(r)$ has a maximum at:

$$\bar{r} = \frac{1}{\sqrt{2Sw}} \qquad (1.5)$$

which yields an indication of the string breaking distance. Models for hadronic processes based on similar ideas exist in the literature (see e.g. [19]).

2. Explicit models

We shall assume for the average chromoelectric field $\mathcal{E}$ the rms value of the chromoelectric field on the cross section of the tube:

$$\mathcal{E} = \sqrt{\langle \mathcal{E}^2 \rangle}\,,$$
$$\langle \mathcal{E}^2 \rangle = \frac{1}{S}\int d^2z\, \mathcal{E}^2(z)\,.$$

At the border between type-I and type-II supeconductor (the so-called Bogomol'nyi limit [20]), $\langle \mathcal{E}^2 \rangle$ can be computed explicitely. The flux tube is the solution [8] of the dual Landau-Ginzburg equation, obtained from the lagrangian:

$$\mathcal{L} = -\frac{1}{4}F^2 + |D\varphi|^2 - V(\varphi)\,. \tag{2.1}$$

Here $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $D_\mu\varphi = (\partial_\mu - i g_m B_\mu)\varphi$, $V(\varphi) = \frac{\lambda}{2}\left(|\varphi|^2 - v^2\right)^2$, B_μ is the dual gauge field, g_m is the magnetic charge and, in the broken phase, $v \neq 0$. The Bogomol'nyi limit corresponds to $m_H = m_V = \sqrt{2} g_m\, v$. In this limit, the solutions to the equations of motion are known [21] and the string tension, defined as the integral of the energy density $\mathcal{H}$ over transverse directions, can be computed exactly [20]:

$$\sigma = \int d^2z\, \mathcal{H}(z) = \frac{\pi m_V^2}{g_m^2}\,. \tag{2.2}$$

Using the Dirac quantization condition, $g g_m = 2\pi$, eq. (2.2) becomes $\sigma r_\perp^2 = \alpha_s$.

Using the solution of ref. [21] for a tube carrying one unit of flux, we compute the average of the electric field squared:

$$\langle \mathcal{E}^2 \rangle = C_{\mathrm{Bog}}\frac{m_V^2\sigma}{\pi}\,, \tag{2.3}$$

where $C_{\mathrm{Bog}} \simeq 0.18$. The exponent in eq. (1.3) becomes then

$$-\ln y \equiv \frac{\pi^2 m^2}{\sqrt{0.18}\, g g_m \sigma}\,, \tag{2.4}$$

and if $g g_m = 2\pi$, as expected in a dual superconductor, then $y \simeq 0.7$. The sum of the terms of order higher than one in eq. (1.3) can be estimated to be $\simeq y^2/4$, that for the worst case of $m = m_\pi$ allows one to keep only the first term in the sum with an accuracy of $\sim 17\%$. The coefficient w can finally be written as:

$$w = N_c N_f \frac{\alpha_s}{\pi^2}\,\frac{0.18}{S}\,\sigma\, e^{-\pi m^2/2\sqrt{0.18}\sigma}\,. \tag{2.5}$$

Note that the potential in this analysis is determined up to a constant term, depending on the physics at shorter distances. Furthermore, $V(r)$ only depends on the string tension σ and the mass m. The potential obtained by inserting eq. (2.5) into eq. (1.4) can be checked on the lattice; the dependence on m^2 can be tested by comparing the potential while varying the bare quark mass.

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Equation (1.5) yields the following expression for the position of the maximum:

$$\bar{r} = \frac{\pi}{\sqrt{0.36 N_c N_f \alpha_s \sigma y}} \,. \tag{2.6}$$

Note that since $\alpha_s N_c$ is constant at large N_c, $\bar{r}$ is N_c-independent in this limit. Next, using $\alpha_s(1\,\mathrm{GeV}) \simeq 0.5$, $N_c = 3$, $N_f = 2$, one obtains from eq. (2.6) the following estimates $\bar{r}(m = 200\,\mathrm{MeV}) \simeq 2.0$ fm, $\bar{r}(m = 300\,\mathrm{MeV}) \simeq 3.3$ fm. There is a degree of arbitrariness in the choice of the scale at which α_s should be computed, so that the latter has to be considered as a parameter in eq. (2.6). In practice [3]–[6], string breaking is detected on the lattice as a deviation of the $\bar{q}q$-potential from the straight line $V = \sigma r$, bigger than the numerical error. Let ε be the value of such a relative error. From eq. (1.4) it follows

$$1 - e^{-r_B^2 S w} \simeq \varepsilon \,,$$

or the breaking distance r_B:

$$r_B = \bar{r}\sqrt{2}\,|\ln(1-\varepsilon)| \,.$$

For instance, for $\varepsilon \simeq 20\%$, $r_B \simeq 0.28\bar{r}$.

The existence of the maximum of the potential and the dependence of its position on m can also be tested directly in numerical simulations. In all this analysis the flux tube is assumed to be that of the quenched theory. Production of gluon pairs is neglected, because the threshold would be given by the lightest glueball mass, which is much larger than the typical hadron mass. This is also the reason why at the one-loop order the formula (1.2) can be translated to QCD in the way this has been done in eq. (1.3).

The same analysis can be performed in the extreme type-II regime (else called the London limit), $\ln\kappa \gg 1$, where $\kappa \equiv m_H/m_V$ is the Ginzburg-Landau parameter with $m_H = \sqrt{2\lambda}v$, $m_V = \sqrt{2}g_m v$. The only difference of this limit from the Bogomol'nyi one is that the coefficient $1/\sqrt{0.18} \simeq 2.36$ in eq. (2.4) should be replaced by $1.18\ln\kappa$. Indeed, in the London limit eq. (2.3) is replaced by

$$\langle \mathcal{E}^2 \rangle = \frac{C_{\mathrm{Lond}}}{\ln\kappa}\frac{m_V^2\sigma}{\pi} \,, \tag{2.7}$$

where $C_{\mathrm{Lond}} \simeq 0.71$. For reasonable values of $\ln\kappa$ (for instance, $\ln\kappa \sim 4$), there is no significant change with respect to the Bogomol'nyi case.

3. Temperature dependence

More information can be extracted from eq. (1.5) if one considers the long-range potential at finite temperature, as the deconfinement temperature is approached.

If the mass in eq. (1.3) is the constituent quark mass, it stays finite at the transition, while the string tension vanishes with a critical exponent:

$$\sigma \sim \left(1 - \frac{T}{T_c}\right)^{\nu} \tag{3.1}$$

where an effective exponent $\nu = 1/3$ can be used for a first order phase transition. In this case, $\bar{r}$ behaves as $\frac{e^{\mathrm{const}/\sigma}}{\sqrt{\sigma}}$. The approximation in eq. (2.5) is improved as we approach the phase transition.

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If instead the mass in eq. (1.3) is the pion mass, it vanishes at the chiral point as:

$$m^2 \sim \left(1 - \frac{T}{T_c}\right)^{\gamma}, \tag{3.2}$$

where the exponent γ is determined by the universality class of the transition. For instance, if the symmetry breaking is described by an $O(4)$ RG fixed point [22], $\gamma \simeq 1.44$. In this case, y approaches one at the critical temperature, and the string breaking distance grows as $1/\sqrt{\sigma}$ as the temperature is increased. One should also remark that eq. (2.2) implies that the transverse size of the flux tube $r_\perp$ is inversely proportional to $\sqrt{\sigma}$. When approaching T_c, $r_\perp^2$ goes large as $(1 - T/T_c)^{-\nu}$. Since $\nu = 1/3$, $r_\perp^2 m^2 \sim (1 - T/T_c)^{1.1}$ goes to zero, implying that the Schwinger formula can no longer be applied when T is very near T_c. These two scenarios can be checked against data from lattice simulations.

A qualitative comparison with the data of ref. [7] for the string breaking length at finite temperature is roughly consistent with the behaviour predicted by combining eq. (2.6) with eqs. (3.1) and (3.2). A more quantitative analysis requires more precise and systematic numerical simulations.

4. Concluding remarks

We have developed a model for string breaking based on the existence of chromoelectric flux tubes. The form of the long-range potential is predicted, which leads to an estimate of the string breaking length; the relevant parameters are the string tension and the mass m, which is to be determined phenomenologically. A prediction is also obtained for the behaviour of the string breaking length near the deconfinement phase transition. Our predictions will be tested against the results of lattice simulations. In particular, the quark masses in lattice simulations can be varied, and m can be understood as a function of them.

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